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A DIGITAL METHOD FOR NOISE REDUCTION IN HOLOGRAPHIC RECONSTRUCTIONS AND ELECTRON MICROSCOPICAL IMAGES

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Abstract

A considerable reduction of noise can be achieved by converting a noisy image into a complex signal (or equivalently into a phase-modulated carrier fringe pattern) and processing this signal (fringe pattern) in the same manner as in the reconstruction step of off-axis holography. A filtered version of the image can be obtained by performing a Fourier transformation of the complex signal (or fringe pattern), applying a digital filter of appropriate size for noise reduction, inverse Fourier transformation of the filtered data, and displaying the phase of the complex result as a reconstructed image. The method is, in particular, applicable to phase images with discontinuities or phase steps. The sharp edges at the steps remain nearly unaffected, and the phase data can therefore be easily unwrapped without falsification of the result. As demonstrated by experimental results, the performance of the proposed method is superior to that of usual filtering methods.

Key Words: Noise reduction, noisy image, fringe pattern, Fourier transformation, phase image, phase unwrapping.

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Introduction

The wide availability of digital image-processing systems has led to the development of a large number of techniques for noise reduction, such as low-pass filtering and median filtering, which are devised for use either in the Fourier domain or directly in real space. Noise represents in general a serious problem in electron microscopy and holography since it sets a limit on the resolution and leads to complications in subsequent steps for image improvement. This is particularly true in the case of reconstructed phase images obtained by Fourier processing of off-axis holograms [1], by direct evaluation from a single hologram [2], or by means of phase-shifting holography [3] where there is a need for unwrapping the data so that accurate calculations of the phase profile can be made; for details on special techniques for phase unwrapping, cf. e.g. [4].

Unwrapping of noisy images would normally introduce a large amount of errors, however, and application of the widely used low-pass filters or median filters to remove the noise in discontinuous phase images is not well-suited, because the first group would affect the sharp edges at the discontinuities, and the latter does not always provide satisfactory and reliable results.

A simple digital method for processing noisy images without blurring sharp edges is now presented, of which the performance is found to be better than that of usual filtering methods. The new method involves in fact low-pass filtering, not of the image itself but of a complex representation of the image. In the following, the principle of the method is explained, and experimental results are given.

Principle

The new technique is based on converting the discontinuous noisy image $\Phi_n(\mathbf{x})$ with a wide Fourier spectrum into a complex signal

$$F_n(\mathbf{x}) = \exp[i\Phi_n(\mathbf{x})];\ \mathbf{x} = (x, y) \qquad (1)$$

with a relatively narrow spectrum. If the image-processing system employed does not allow such a conversion, the

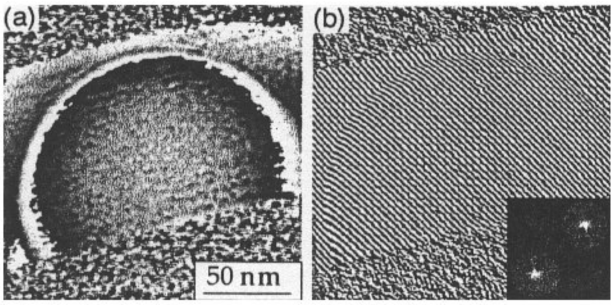

Figure 1. Noisy image (**a**) and corresponding computer-generated fringe pattern (**b**) with its Fourier spectrum as inset. The noisy image was obtained by holographic phase reconstruction from an off-axis-hologram of a latex sphere.

noisy image can equivalently be converted into a fringe pattern of the form

$$\begin{aligned} P(\mathbf{x}) &= u + 2\cos\{2\pi\mathbf{R}_c\mathbf{x} - \Phi_n(\mathbf{x})\} \\ &= u + F_n(\mathbf{x})\exp(-i2\pi\mathbf{R}_c\mathbf{x}) \\ &\quad + F_n^*(\mathbf{x})\exp(+i2\pi\mathbf{R}_c\mathbf{x}) \end{aligned} \qquad (2)$$

in which the information on the image is encoded in the location of the fringes (Fig. 1). In Equation (2), the symbol u represents a constant background and $|\mathbf{R}_c| = R_c$ is the carrier frequency, which is defined as the reciprocal of the fringe distance. Comparison of Equation (2) with the expression

$$H(\mathbf{x}) = 1 + [A(\mathbf{x})]^2 + 2A(\mathbf{x})\cos\{2\pi\mathbf{R}_c\mathbf{x} - \Phi(\mathbf{x})\} \qquad (3)$$

for an off-axis hologram, where $A(\mathbf{x})$ and $\Phi(\mathbf{x})$ represent the amplitude and phase of the object function, shows that the new approach is closely related to the method of holography employing pure phase objects (i.e., $A(\mathbf{x}) = 1$) (cf., e.g., Section II.A in [1]). A filtered version of the image with considerably reduced noise can be obtained from the fringe pattern (2), or from the complex signal (1), by performing the same procedure as in the holographic reconstruction step. This procedure consists mainly of Fourier-transformation (FT), filtering, inverse Fourier transformation (FT^{-1}), and displaying the result as a reconstructed image. A brief description of these operations for the case of a carrier fringe pattern will now be given (Fig. 2).

Fourier transformation

In the first step, the fringe pattern is Fourier-transformed to obtain the corresponding spectrum. A two-dimensional Fourier transformation of (2) leads directly to the result

$$\begin{aligned} \tilde{P}(\rho) &= u\,\delta(\boldsymbol{\rho}) && \langle 0 \rangle \\ &\quad + \tilde{F}_n(\mathbf{R}_c + \boldsymbol{\rho}) && \langle -1 \rangle \\ &\quad + \tilde{F}_n^*(\mathbf{R}_c - \boldsymbol{\rho}) && \langle +1 \rangle \end{aligned} \qquad (4)$$

where the tilde has been used to indicate the Fourier transform of the function under consideration. Besides the zero-order term <0>, which represents the constant background in (2), two laterally shifted sidebands, <-1> and <+1>, representing the Fourier transform of the complex image function $F_n(\mathbf{x})$ and its conjugate are found in the spectrum (Fig. 1b). [*Note*: in the case of the complex signal (1), the spectrum consists only of the unshifted sideband <-1>.]

Reconstruction of a filtered image

The information on the noisy image is stored in the aforementioned sidebands. To reconstruct the desired filtered image, it is therefore sufficient to use only one of them. To isolate the sideband of interest and to eliminate unwanted parts of the spectrum, a digital filter (e.g., Gaussian) of appropriate size is employed. The filtered sideband is then shifted to the origin of the Fourier space and an inverse Fourier transformation of the corresponding data is performed. [*Note* that if the signal (1) is used instead of the fringe pattern, no shifting in Fourier space is required.]

To avoid a great loss of resolution, the carrier frequency R_c should be made sufficiently large. If the width of the Fourier space is denoted by $2R_{max}$, the carrier frequency R_c can, in general, be chosen to be equal to one half of R_{max}. This allows the reconstruction of filtered image signals with resolutions up to the limit $R_{max}/2$. To reduce the effect of noise, however, the radius of the digital filter employed is generally made smaller than this limit. [*Note*: when the complex signal (1) is processed, the resolution is given by R_{max}.] Hence, the final resolution is ultimately limited by the actual size of the filter.

Calculation of amplitude and phase

Inverse Fourier transformation of the filtered sideband results in a complex function $F(\mathbf{x})$ which can be analyzed by displaying its amplitude

$$a(\mathbf{x}) = \{[\mathrm{Re}(F)]^2 + [\mathrm{Im}(F)]^2\}^{1/2} \qquad (5)$$

and phase

$$\Phi(\mathbf{x}) = \arctan[\mathrm{Im}(F)/\mathrm{Re}(F)] \qquad (6)$$

as reconstructed amplitude and phase images, where $\mathrm{Re}(F)$ and $\mathrm{Im}(F)$ represent the real and imaginary part of the function $F(\mathbf{x})$, respectively. The signs of $\mathrm{Re}(F)$ and $\mathrm{Im}(F)$ must be taken into account to obtain phase values ranging from $-\pi$ to $+\pi$. The resulting phase values are thus always

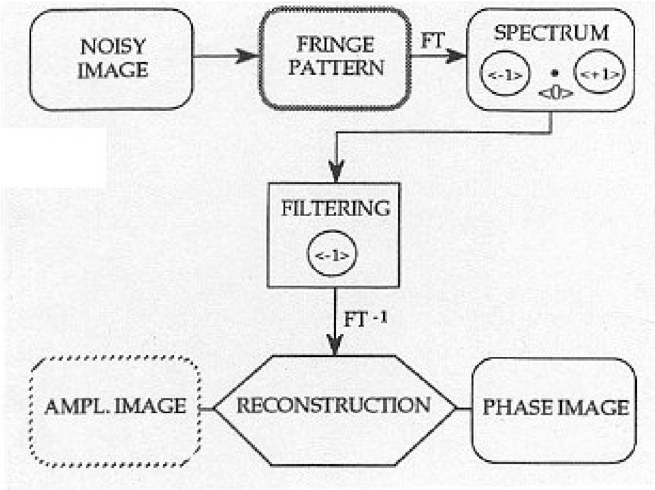

Figure 2. Schematic diagram of the digital procedure for noise reduction.

modulo 2π.

Note that if the sideband used for reconstruction is not shifted to the origin of the Fourier space, a linear phase ramp of the form

$$F_{ramp} = -2\pi\mathbf{R_c x} \tag{7}$$

will be added to the phase image of interest, as a consequence of the shift theorem of Fourier transforms.

In the following we are mainly interested in the phase image given by means of Equation (6). As stated above, the quality and the resolution of this image depend greatly on the size of the digital filter employed in the spectrum. If the filter size is large enough, the reconstructed image will agree with the original image. By reducing the filter size, however, unwanted spectral parts and a large amount of the random noise present can be removed. To avoid complications arising from artifacts and streaking effects that might occur in the spectrum, it is recommended to use fringes with an inclination of $|\pi/4|$ with respect to the coordinate axis.

Experimental Results

To demonstrate the performance of the proposed method, two experimental examples will now be given. The off-axis holograms used for obtaining the noisy images were produced in the electron microscope (Philips EM 400T with field-emission gun; Philips Electron Optics, Eindhoven, The Netherlands) and recorded on photographic plates. Subsequently, the holograms were scanned with a TV camera, and the video signals were digitized in a 512x512 raster with an 8-bit grey scale by means of a TEMDIPS image-processing system (Tietz Video and Image Processing

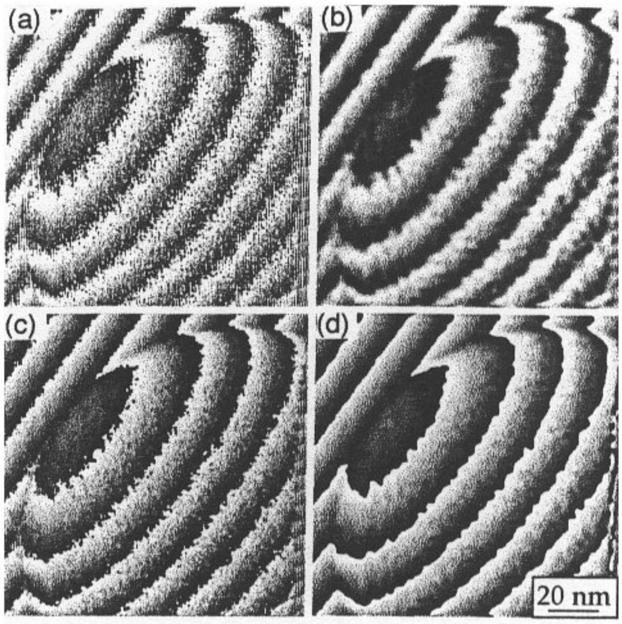

Figure 3. Phase images of a latex sphere. (**a**): Image reconstructed by direct phase-evaluation [2]. (**b**)-(**d**): Images obtained from the result (a) by employing a low-pass filter, a 5x5 pixel median filter, and the new filtering technique, respectively. The filter size used in (d) is the same as in (b).

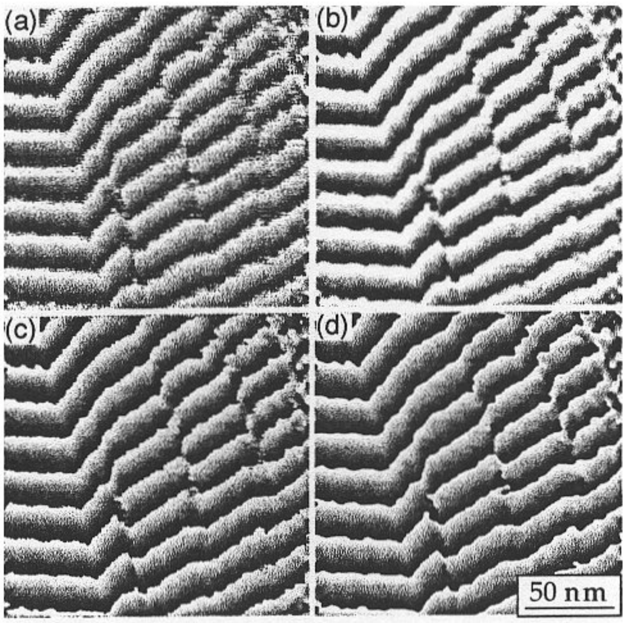

Figure 4. Phase images of a silicon wedge revealing dynamical phase jumps of π. (**a**): Holographically reconstructed image. (**b**)-(**d**): Filtered images corresponding to those in Figure 3. The filter size used in (d) is the same as in (b).

Systems, Gauting, Germany).

In the first example, a latex sphere is chosen as an object. The noisy image to be processed (Fig. 3a) was obtained from the corresponding off-axis hologram by employing the direct phase-evaluation method described in [2] and adding a linear phase ramp to the result. The fringes indicate positions of equal phase, and the bends of the fringes indicate local thickness variations. Thus, the thickness profile of the sphere is made visible by means of the fringes occurring at the locations of phase discontinuities or steps. To get rid of the noise, a median or a low-pass filter is usually applied to the noisy image. The results obtained by employing such filters are shown in Figures 3b and 3c. Both images are still found to be very noisy, and the edges are unsharp. These problems do not arise in the new filtering technique described above. As demonstrated in Figure 3d, where the filter size employed is the same as in Figure 3b, the filtered image shows very sharp edges and is free from disturbing noise.

These statements apply also to the results of the second example shown in Figure 4. The noisy image of Figure 4a represents a holographic phase reconstruction of a silicon wedge revealing dynamic phase jumps of π; for more details cf. Section IV.B in [1]. The reconstruction process was performed in the usual manner, and the resulting phase distribution was superposed on a linear phase ramp. In contrast to median and low-pass filtering, where the edges are unsharp and the noise is only partially eliminated (Figs. 4b,c), the new technique leads to a nearly noise-free-image without affecting the sharp edges at the steps (cf. Fig. 4d)

Summary

We have investigated the possibility of digitally processing noisy images to reduce the effect of noise. It has been shown that a considerable reduction of noise can be achieved by converting the noisy image into a complex signal or, equivalently, into a carrier fringe pattern and processing this signal (or pattern) in a similar manner as in the reconstruction step of off-axis holography. As demonstrated by experimental results, the performance of the method is better than that of usual filtering methods. Furthermore, the method is especially well-suited for processing noisy images with sharp edges such as holographically reconstructed phase images.

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